Enrollment No: ______________________ **Exam Seat No:**______________________

C.U.SHAH UNIVERSITY

Summer Examination-2016

Subject Name : Engineering Mathematics-I
Subject Code : 4TE01EMT1

Branch : B.Tech(All)

Semester : 1 **Date :21/04/2016** **Time :10:30 To 1:30** **Marks : 70**

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 **Attempt the following questions:** **(14)**

a) If $z = re^{i\theta}$, then $\left|re^{iz}\right| =$

a) $e^{r\sin\theta}$ b) $e^{-r\sin\theta}$ c) $e^{-r\cos\theta}$ d) $e^{r\cos\theta}$

b) The Imaginary part of Complex number e^{3z} is

a) $e^{y}\sin x$ b) $e^{x}\cos y$ c) $e^{3x}\cos 3y$ d) $e^{3x}\sin 3y$

c) $\lim_{x\to 0}\frac{\cos x}{x} =$ _____.

a) 0 b) 1 c) ∞ d)-1

d) $\lim_{x\to 0}\frac{x-\sin x}{x} =$ _____.

a) 0 b) 1 c) ∞ d)-1

e) The series $\sum\frac{1}{n}$ is

a) Convergent b) Divergent c) non-convergent d) a & b both

f) The series $\sum_{n=1}^{\infty}\frac{3n-5}{11n^2+2}$ is

a) Convergent b) Divergent c) non-convergent d) None of these

g) If the power of x & y both are even ,then the curve is symmetrical about

a) X-axis b) Y-axis c)about both X & Y axes d)None of these

h) If the two tangents at the point are real & distinct, the double point is called

a) a node b) a cusp c) a conjugate point d)None of these

i) The series $x-\frac{x^3}{3!}+\frac{x^5}{5!}-.....$ represents expansion of

a) $\sin x$ b) $\cos x$ c) $\cosh x$ d) $\sinh x$

j) If $y = \cos^{-1} x$, then x=……

a) $1-\frac{y^2}{2!}+\frac{y^4}{4!}-....$ b) $1+\frac{y^2}{2!}+\frac{y^4}{4!}+....$ c) $y-\frac{y^3}{3!}+\frac{y^5}{5!}-....$ d) None of these

k) If $u=\sin^{-1}\left(\dfrac{x^2+y^2}{x+y}\right)$ then $x\dfrac{\partial u}{\partial x}+y\dfrac{\partial u}{\partial y}=.....$

a) u b) 2u c) tan u d) sin u

l) If $u=y^x$ then $\dfrac{\partial u}{\partial x}$ is

a) xy^{y-1} b) 0 c) $y^x \log x$ d) None of these

m) If $x=r\cos\theta, y=r\sin\theta, z=z$ then $\dfrac{\partial(x,y,z)}{\partial(r,\theta,z)}=....$

a) $\dfrac{1}{r}$ b) $r^2\sin\theta$ c) r d) $r^2\cos\theta$

n) If $\dfrac{\partial(u,v)}{\partial(x,y)}*\dfrac{\partial(x,y)}{\partial(u,v)}=....$

a) 0 b) -1 c) 1 d) None of these

Attempt any four questions from Q-2 to Q-8

Q-2 **Attempt all questions** **(14)**

A i) Find modulus and principal argument of $z=\dfrac{1-7i}{(3+4i)}$. **(03)**

ii) Solve the equation $z^2-(5+i)z+8+i=0$. **(04)**

B Find and plot all the roots of $(1+i)^{\frac{1}{3}}$. **(07)**

Q-3 **Attempt all questions** **(14)**

A i) Evaluate: $\lim\limits_{x\to 0}\left[\dfrac{a^x+b^x+c^x}{3}\right]^{\frac{1}{x}}$. **(04)**

ii) Find Maclaurin's Series of $f(x)=\cos x$. **(03)**

B i) Show that $f(x,y)=\begin{cases}\dfrac{2xy}{x^2+y^2}; & (x,y)\neq(0,0)\\ 0 \quad ; & (x,y)=(0,0)\end{cases}$ is continuous at every point except at the origin. **(04)**

ii) Evaluate: $\lim\limits_{x\to 0}\dfrac{\tan x-x}{x^2\tan x}$.. **(03)**

Q-4 **Attempt all questions** **(14)**

A Trace the curve **(Cissoid of Diocle)** $y^2(2a-x)=x^3$. **(07)**

B Find the Taylor's series expansion of $f(x)=\tan x$ in powers of $\left(x-\dfrac{\pi}{4}\right)$ showing at four nonzero terms. Hence, find the value of $\tan 45^0$. **(07)**

Q-5 **Attempt all questions** **(14)**

A Trace the curve **(Cardioid)** $r = a(1+\cos\theta)$. **(07)**

B

i) Test the convergence of the series $\frac{2}{1}+\frac{3}{8}+\frac{4}{27}+........+\frac{n+1}{n^3}+.....$ **(04)**

ii) Test the convergence of the series **(03)**

$$\frac{1}{1.3}+\frac{1}{3.5}+\frac{1}{5.7}+........+\frac{1}{(2n-1).(2n+1)}+.....$$

Q-6 **Attempt all questions** **(14)**

A

(i) Test the convergence of the series $\frac{1}{1+3}+\frac{2}{1+3^2}+\frac{3}{1+3^3}+........$ **(05)**

(ii) Test the convergence of the series $\sum_{n=1}^{\infty}\frac{1}{n^{\frac{3}{2}}}$. **(02)**

B Find the radius of convergence & interval of convergence of the series **(07)**

$$\sum_{n=0}^{\infty}\frac{(-3)^n x^n}{\sqrt{n+1}}.$$

Q-7 **Attempt all questions** **(14)**

A

(i) If $u=\log\left(x^3+y^3-x^2y-xy^2\right)$. then Prove that $\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}\right)^2 u=-\frac{4}{(x+y)^2}$. **(05)**

(ii) Find the values of $\frac{\partial f}{\partial x}$ & $\frac{\partial f}{\partial y}$ at the point (4,-5).if $f(x,y)=x^2+3xy+y-1$. **(02)**

B

(i) If $u=\sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$. Prove that **(07)**

a) $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\frac{1}{2}\tan u$

b) $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=\frac{1}{4}\left(\tan^3 u-\tan u\right)$.

Q-8 **Attempt all questions** **(14)**

A

(i) Find Maxima & Minima of the function $x^3+y^3-3x-12y+20$. **(05)**

(ii) If $x=r\cos\theta, y=\sin\theta$ then find $\frac{\partial(x,y)}{\partial(r,\theta)}$. **(02)**

B

(i) Expand $f(x,y)=e^x\cos y$ in powers of x & y up to second degree. **(05)**

(ii) Find the equations of tangent plane & normal line at the point (-2,2,-3) to the ellipsoid $\frac{x^2}{4}+y^2+\frac{z^2}{9}-3=0$ **(02)**